



# Test Fixture Effects in Vibration Tests of Rocket Motors

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Calspan Corporation, AEDC Division

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November 1985

Final Report for Period November 1, 1982 – May 1, 1983

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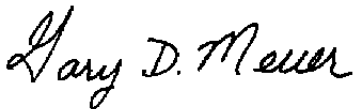
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SECURITY CLASSIFICATION OF THIS PAGE

## REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION <b>UNCLASSIFIED</b>			1b. RESTRICTIVE MARKINGS		
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION/AVAILABILITY OF REPORT		
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE			See Reverse of This Page.		
4. PERFORMING ORGANIZATION REPORT NUMBER(S)  AEDC-TR-85-42			5. MONITORING ORGANIZATION REPORT NUMBER(S)		
6a. NAME OF PERFORMING ORGANIZATION Arnold Engineering Development Center		6b. OFFICE SYMBOL (If applicable)  DOF		7a. NAME OF MONITORING ORGANIZATION	
6c. ADDRESS (City, State and ZIP Code) Air Force Systems Command Arnold Air Force Station, TN 37389-5000				7b. ADDRESS (City, State and ZIP Code)	
8a. NAME OF FUNDING/SPONSORING ORGANIZATION Arnold Engineering Development Center		8b. OFFICE SYMBOL (If applicable)  DO		8. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	
8c. ADDRESS (City, State and ZIP Code) Air Force Systems Command Arnold Air Force Station, TN 37389-5000				10. SOURCE OF FUNDING NOS.	
11. TITLE (Include Security Classification) See Reverse of This Page.				PROGRAM ELEMENT NO.  65807F	TASK NO.
12. PERSONAL AUTHOR(S) Welsh, C. J., Calspan Corporation, AEDC Division					
13a. TYPE OF REPORT Final Report		13b. TIME COVERED FROM 11/1/82 TO 5/1/83		14. DATE OF REPORT (Yr., Mo., Day) November 1985	
15. PAGE COUNT 24					
18. SUPPLEMENTARY NOTATION Available in Defense Technical Information Center (DTIC).					
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB. GR.	rocket motors		
21	08	.2	vibration testing		
20	11		test fixtures		
19. ABSTRACT (Continue on reverse if necessary and identify by block number)  An analysis was made of acceleration response measurements of solid-propellant rocket motors in vibration testing at the Arnold Engineering Development Center. In this type of testing, an electrodynamic shaker is utilized to provide the desired oscillatory driving force, and a test fixture is required to adapt the motor to the shaker. Although a test fixture can contribute adverse effects on the rocket motor motions for some conditions in vibration testing, the results of the analysis indicate that large undesirable rocket motor acceleration responses may be measured and invalidly attributed to the test fixture used.					
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT  UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT. <input checked="" type="checkbox"/> DTIC USERS <input type="checkbox"/>			21. ABSTRACT SECURITY CLASSIFICATION  UNCLASSIFIED		
22a. NAME OF RESPONSIBLE INDIVIDUAL  W.O. Cole			22b. TELEPHONE NUMBER (Include Area Code) (615)454-7813		22c. OFFICE SYMBOL  DOS

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## **PREFACE**

The work reported herein was performed by the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC). The results were obtained by Calspan Corporation, AEDC Division, operating contractor for the aerospace flight dynamics testing effort at the AEDC, AFSC, Arnold Air Force Station, Tennessee under Project Number V44W83A. The Project Monitor was Lt. Gary Meuer. The research was performed from November 1, 1983 through May 1, 1983, and the manuscript was submitted for publication on April 24, 1985.

## CONTENTS

	<u>Page</u>
1.0 INTRODUCTION .....	5
2.0 DISCUSSION .....	5
2.1 Procedure in Vibration Testing .....	5
2.2 Identification of the Problem of Concern .....	6
2.3 Motion Equations for a Two-DOF System .....	8
2.4 Experimental Data for a Two-DOF System .....	11
2.5 Rocket Motor Measurements .....	15
3.0 CONCLUDING REMARKS .....	19

## ILLUSTRATIONS

### Figure

1. Typical Electrodynamic Shaker, Test Fixture, and Rocket Motor Assembly .....	6
2. Representative Acceleration Response Curve for a Rocket Motor— A-Test Program .....	7
3. Acceleration Curve for the Test Fixture—A-Test Program .....	7
4. Simplified Rocket Motor Vibration System and Corresponding Forces Involved .....	9
5. Two-Degree-of-Freedom Vibration System Experiment .....	12
6. $P_0$ and $P'_0$ Measurements .....	12
7. Acceleration Response Curve for the Second Mass of the Two-Degree- of-Freedom System Experiment .....	13
8. Inputs for the Two-Degree-of-Freedom System Experiment .....	14
9. Acceleration Curve for a Typical Armature-Fixture Combination .....	15
10. Inputs for Armature-Fixture Test of Fig. 9 .....	16
11. Inputs Corresponding to the Acceleration Response Curve of Fig. 2 .....	17
 NOMENCLATURE .....	 19

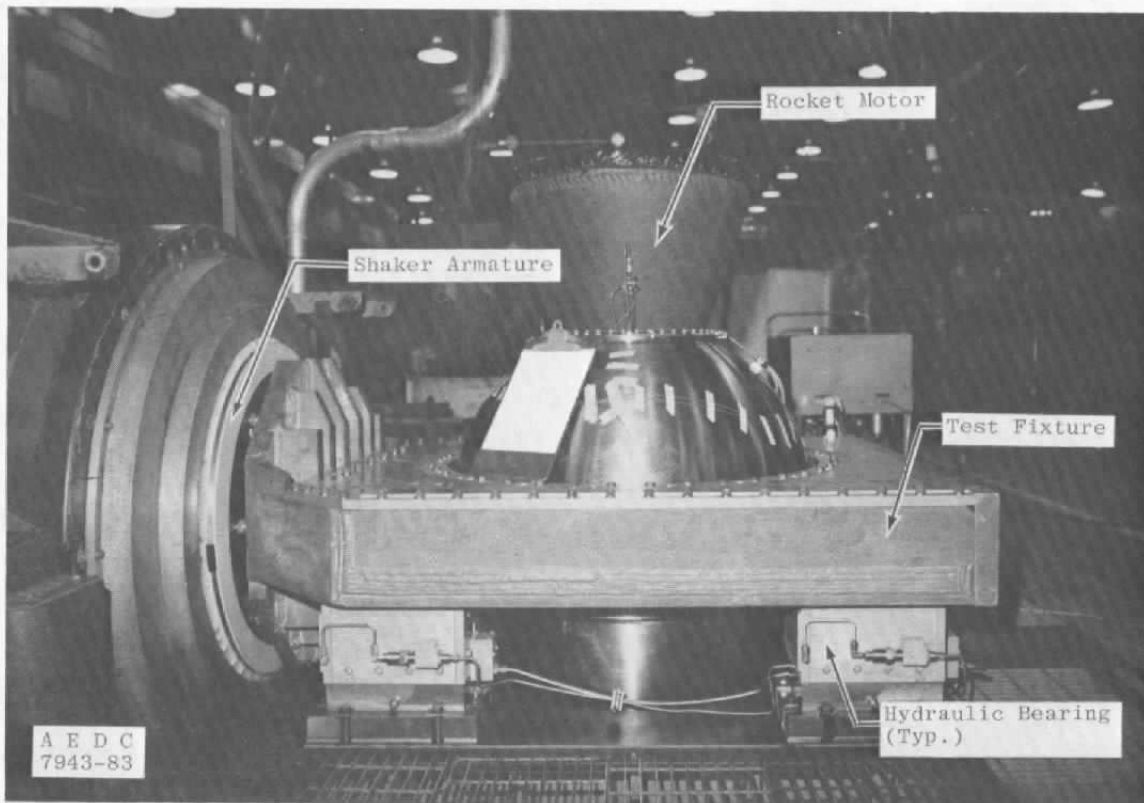
## 1.0 INTRODUCTION

Vibration testing is an important part of any ground test program directed toward the validation of the flight readiness of a solid-propellant rocket motor. Such a motor may be utilized in either an aircraft-launched system or in a multistage booster system. In vibration testing of a rocket motor, the motor is exposed to a vibration environment simulating its anticipated flight environment prior to ignition. In this type of testing, a test fixture is required to permit adapting the rocket motor to an electrodynamic shaker which provides the required forcing function to the fixture-motor combination. The responses of the motor and its components to the ground test vibration environment are evaluated from measurements obtained using accelerometers mounted on the motor and on the test fixture. Anytime that an undesirable motor response is experienced during a test, possible adverse contributions of the fixture become a major concern. Such fixture concerns are related to fixture design problems which arise primarily because of the incompatibility of fixture weight and rigidity constraints. To minimize fixture effects in tests, a more rigid fixture is desired which corresponds, in general, to larger fixture weight. However, allowable fixture weight is normally restricted by the limited forcing capability of the shaker. It follows that required compromises in a fixture design can contribute to adverse characteristics of a test fixture which is normally provided by the ground test facility that is performing the vibration testing. The purpose of this report is to present the results of a study concerned with the interpretation of rocket motor response measurements and to define a means for identifying frequency ranges of potential adverse effects of test fixtures being used.

## 2.0 DISCUSSION

### 2.1 PROCEDURE IN VIBRATION TESTING

In vibration testing of a rocket motor, usually made over a frequency range of 5 to 2,000 Hz, the desired fixture acceleration level is maintained with use of a control system which monitors the outputs of control accelerometers (mounted on the test fixture in appropriate locations) and adjusts the input signal to the shaker consistent with the measured outputs of the control accelerometers. A photograph of a typical rocket motor, test fixture, and electrodynamic shaker system used in vibration testing in the horizontal plane is shown in Fig. 1. Although different modes of vibration testing are utilized, any adverse contributions of a test fixture can be best identified using the sine sweep mode in conjunction with a control system designed to maintain a constant fixture acceleration level over the frequency range. The typical frequency sweep rate is two octaves per minute.

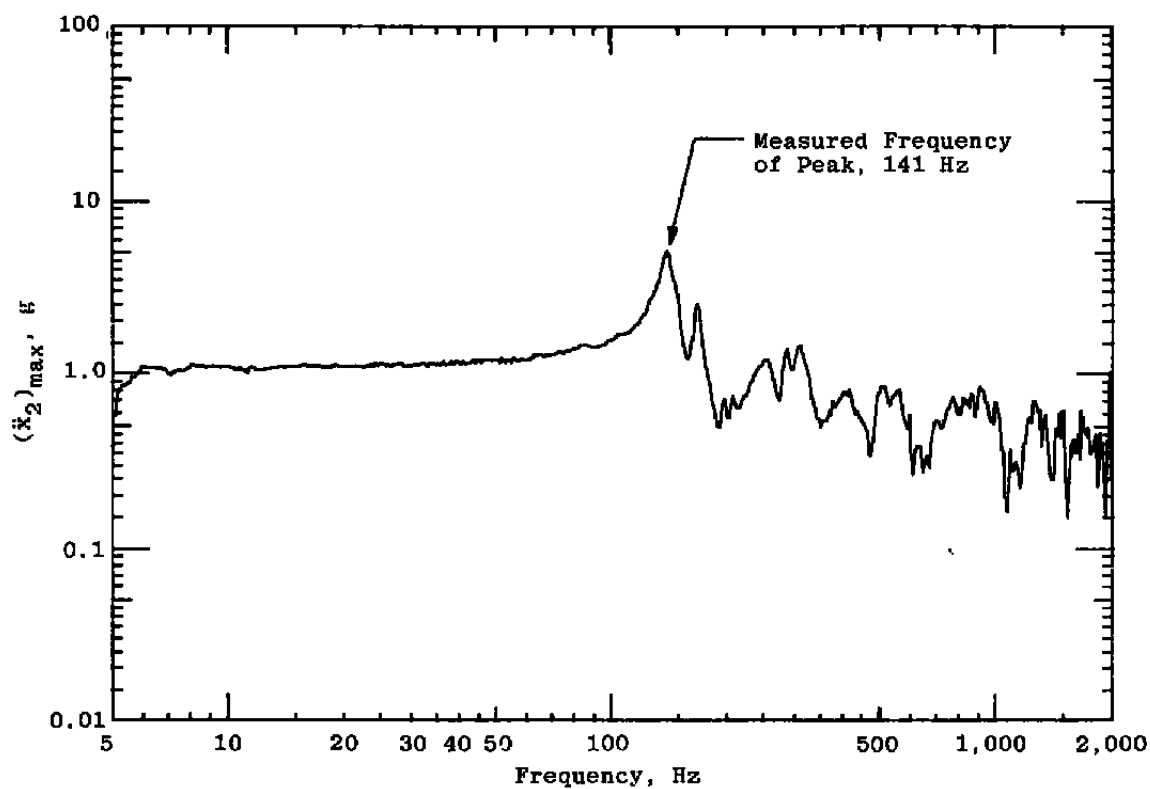


**Figure 1. Typical electrodynamic shaker, test fixture, and rocket motor assembly.**

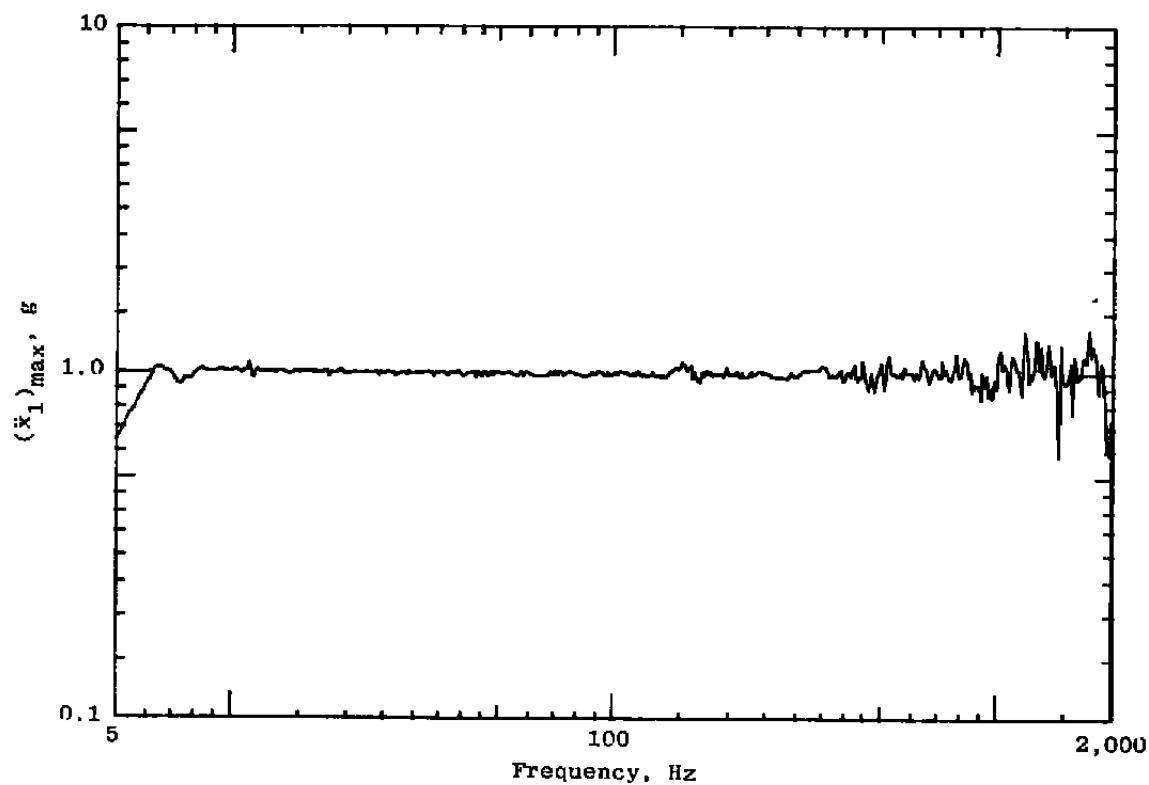
## **2.2 IDENTIFICATION OF THE PROBLEM OF CONCERN**

An acceleration response curve (sine sweep mode) for a rocket motor as a function of the forcing frequency that was obtained in a previous vibration test program, referred to here as the A-test program, is shown in Fig. 2. This curve is for a test performed in the horizontal plane and the corresponding input acceleration curve, the average of measurements from the control accelerometers positioned on the test fixture, is shown in Fig. 3. The large increase in the response acceleration for the motor in the region of 141 Hz caused appreciable concern. The cause for this undesirable response was attributed, at the time of the A-test program, to adverse test fixture effects. Test fixtures used in vibration tests in the horizontal plane are similar to the one shown in Fig. 1. The basic portion of such a fixture is of box-type construction and is mounted on four hydraulic bearings. Regions of high acceleration similar to that shown in Fig. 2 have been observed in tests of other rocket motors, and they normally occur within a frequency range of 100 to 200 Hz.





**Figure 2. Representative acceleration response curve for a rocket motor—A-test program.**



**Figure 3. Acceleration curve for the test fixture—A-test program.**

More recently, this problem area was examined at AEDC on the premise that the predominant motions of the shaker armature, test fixture, and rocket motor system can be adequately defined, below some frequency level, by the motion equations for a forced two-degree-of-freedom (DOF) system. Here, the combination of the test fixture and shaker armature having a high-strength bolted joint corresponds to the first mass ( $m_1$ ) and the rocket motor corresponds to the second mass ( $m_2$ ). The stiffness parameter ( $k_1$ ) for  $m_1$  is provided by the armature support flexures and the stiffness parameter ( $k_2$ ) for  $m_2$  is provided by the rocket motor mounting flange which attaches to the test fixture. Note that different types of motor mounting flanges are utilized. In some flange designs, the  $k_2$  stiffness parameter is completely provided by the flange material, whereas in other designs,  $k_2$  can also be dependent on the array of bolts used in attaching the flange to the test fixture. The results of the examination are discussed herein.

### 2.3 MOTION EQUATIONS FOR A TWO-DOF SYSTEM

To permit clarification of the basic characteristics of a two-DOF vibration system, the motion equations for such a system are listed and discussed. Consistent with the sketches of Fig. 4 showing a simplified rocket motor vibration system and the corresponding free-body forces involved in a forced, zero-damped, two-DOF system, the equations of motion for the first mass,  $m_1$ , and for the second mass,  $m_2$ , can be written

$$\begin{aligned} m_1 \ddot{x}_1 &= -k_1 x_1 + k_2 (x_2 - x_1) + F \\ m_2 \ddot{x}_2 &= -k_2 (x_2 - x_1) \end{aligned}$$

and rearranging terms

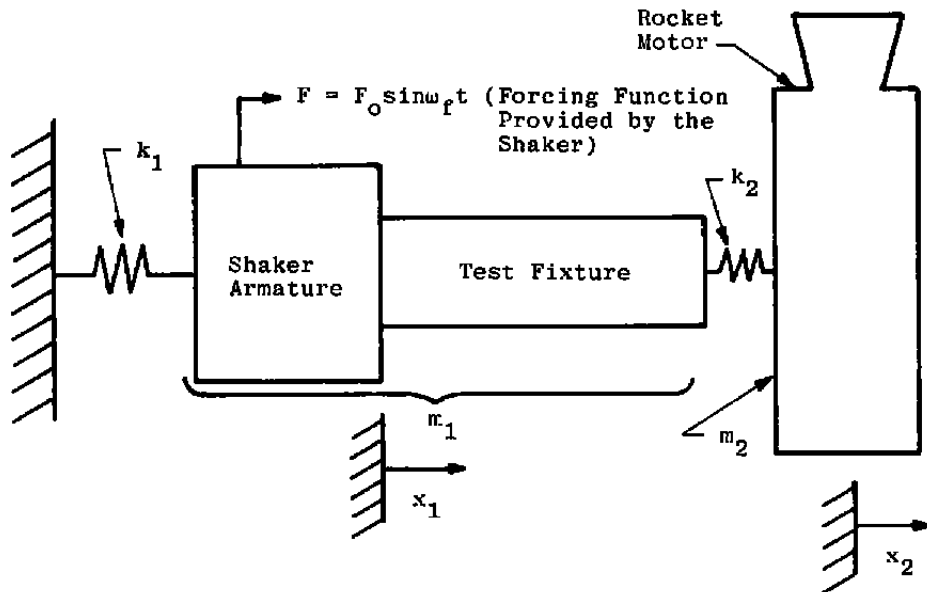
$$m_1 \ddot{x}_1 + (k_1 + k_2)x_1 - k_2 x_2 = F \quad (1)$$

$$m_2 \ddot{x}_2 - k_2 x_1 + k_2 x_2 = 0 \quad (2)$$

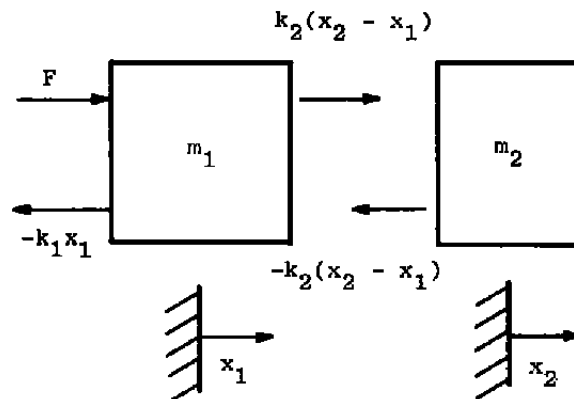
here,  $\ddot{x}_1$  and  $\ddot{x}_2$  are accelerations of  $m_1$  and  $m_2$ , respectively. The particular solutions for these equations, using  $F_0 \sin \omega t$  for the forcing function,  $F$ , can be written

$$x_1 = X_1 \sin \omega t$$

$$x_2 = X_2 \sin \omega t$$



### a. Two-DOF rocket motor vibration system



Notes:  $k_1$  - Stiffness Parameter Provided by the Armature Support Flexures  
 $k_2$  - Stiffness Parameter Between the Rocket Motor and the Test Fixture Resulting from the Motor Attachment Flange  
 $m_1$  - Mass of the Shaker-Armature and the Test Fixture Combination Which Utilizes a High-Strength Bolted Joint  
 $m_2$  - Mass of the Rocket Motor  
 $x_1, x_2$  - Displacements of  $m_1$  and  $m_2$ , respectively

**b. Corresponding free-body diagram**

**Figure 4. Simplified rocket motor vibration system and corresponding forces involved.**

substituting these expressions for  $x_1$  and  $x_2$  into Eqs. (1) and (2),

$$-m_1\omega_f^2 X_1 + (k_1 + k_2) X_1 - k_2 X_2 = F_0$$

$$-m_2\omega_f^2 X_2 - k_2 X_1 + k_2 X_2 = 0$$

and rearranging terms provide the following equations:

$$(-m_1\omega_f^2 + k_1 + k_2) X_1 - k_2 X_2 = F_0 \quad (3)$$

$$(-m_2\omega_f^2 + k_2) X_2 - k_2 X_1 = 0 \quad (4)$$

Incorporating the substitutions

$\omega_1 = \sqrt{k_1/m_1}$ , natural frequency of the decoupled  $m_1$  mass ( $k_2 = 0$ )

$\omega_2 = \sqrt{k_2/m_2}$ , natural frequency of the decoupled  $m_2$  mass ( $m_1$  fixed)

$X_0 = F_0/k_1$ , static deflection of  $m_1$  for a static force  $F_0$ .

into Eqs. (3) and (4), expressions for the maximum displacements,  $X_1$  and  $X_2$  of  $m_1$  and  $m_2$ , respectively, can be written

$$X_1 = \frac{(1 - \omega_f^2/\omega_2^2)X_0}{[(1 + k_2/k_1 - \omega_f^2/\omega_1^2)(1 - \omega_f^2/\omega_2^2) - k_2/k_1]} \quad (5)$$

$$X_2 = \frac{X_0}{[(1 + k_2/k_1 - \omega_f^2/\omega_1^2)(1 - \omega_f^2/\omega_2^2) - k_2/k_1]} \quad (6)$$

or

$$X_2 = X_1/(1 - \omega_f^2/\omega_2^2) \quad (7)$$

Note in Eq. (5) that when the forcing frequency,  $\omega_f$ , is equal to  $\omega_2$  that  $X_1$  tends to go to zero, and from Eq. (6) that

$$X_2 = X_0/(-k_2/k_1) = -F_0/k_2 \quad (7a)$$

These are very significant observations in conjunction with the sine sweep testing mode as again the control system is designed to keep the maximum acceleration of the first mass,  $(\ddot{x}_1)_{\max}$ , at a predefined constant level over the frequency range. Note for a given frequency

that  $(\ddot{x}_1)_{\max}$  and  $(\ddot{x}_2)_{\max}$  are directly proportional to  $X_1$  and  $X_2$ , respectively. Hence, as  $\omega_f$  is swept through  $\omega_2$ , the control system adjusts the shaker input signal to increase  $F_0$  to a peak value at  $\omega_f = \omega_2$  in attempting to keep  $(\ddot{x}_1)_{\max}$  at its predefined level. At this same testing condition, ( $\omega_f = \omega_2$ ), the acceleration of the second mass,  $(\ddot{x}_2)_{\max}$  will increase to a peak value consistent with the peak value for  $X_2$  as defined by Eq. (7a).

Also observe in Eqs. (5) and (6) that for  $\omega_f \neq \omega_2$ , that both  $X_1$  and  $X_2$  will tend to become infinite when the denominator in the expressions for  $X_1$  and  $X_2$  is zero. The corresponding forcing frequency relationship for this condition can be written

$$\omega_f/\omega_2 = \sqrt{\frac{1}{2}[(\omega_1^2/\omega_2^2) + (m_2/m_1) + 1] \pm \sqrt{\left\{\frac{1}{2}[(\omega_1^2/\omega_2^2) + (m_2/m_1) + 1]\right\}^2 - \omega_1^2/\omega_2^2}} \quad (8)$$

The two values of  $\omega_f$  defined by this expression are the natural frequencies of the two-DOF system; the larger value of  $\omega_f$  to satisfy this condition is designated  $\omega_2'$  in this report, and the smaller value is designated  $\omega_1'$ . It follows that for the forcing frequency at either  $\omega_1'$  or  $\omega_2'$  (again both  $X_1$  and  $X_2$  are experiencing resonant conditions at these frequencies) that the shaker force,  $F_0$ , required will necessarily be at a local minimum level.

The more important observations from the above discussion are that the motion equations for a forced two-DOF system with a control system designed to maintain the first mass at a constant acceleration level over the frequency range indicate (1) at  $\omega_f = \omega_2$ , there will be detectable peaks in both the  $(\ddot{x}_2)_{\max}$  and  $F_0$  curves, and (2) there will be detectable local minimums in the  $F_0$  curve at  $\omega_f = \omega_1'$  and at  $\omega_f = \omega_2'$ .

## 2.4 EXPERIMENTAL DATA FOR A TWO-DOF SYSTEM

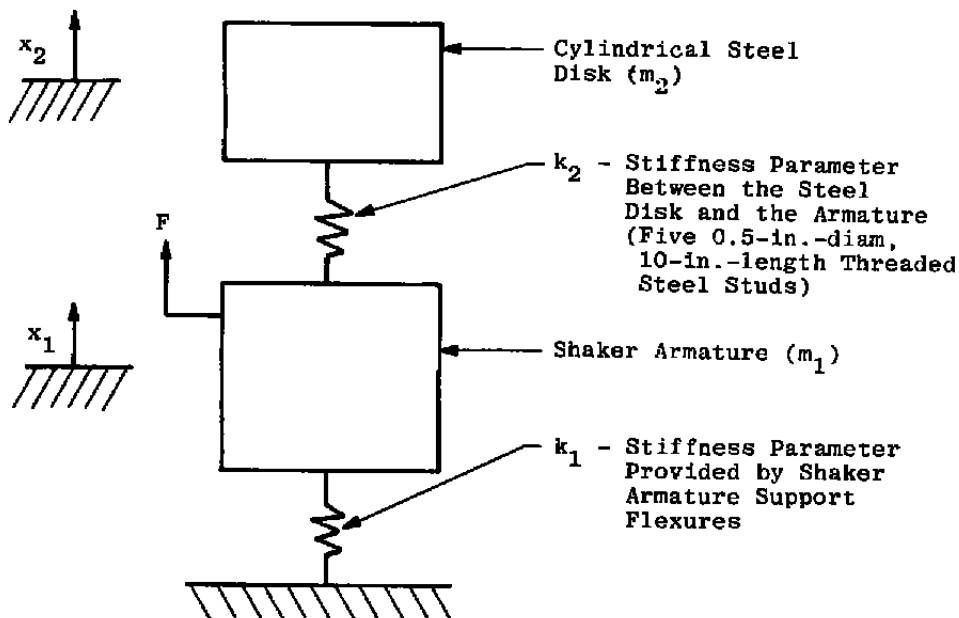
To demonstrate the usefulness of the motion equations in examining experimental two-DOF systems, a limited amount of experimental data using the sine mode of testing was obtained in an experiment for the near-ideal two-DOF system defined in Fig. 5. Measured input signals for the system ( $P_0$  and  $P_0'$ ) are shown in Fig. 6. Using the listed values in Fig. 5 for  $m_1$ ,  $m_2$ ,  $k_1$ , and  $k_2$ , the following calculated values for the natural frequency of the decoupled second mass,  $\omega_2$ , and the natural frequencies  $\omega_1'$  and  $\omega_2'$  of the two-DOF system were obtained, using Eq. (8),

$$\omega_2 = 137.8 \text{ Hz}$$

$$\omega_1' = 8.7 \text{ Hz}$$

$$\omega_2' = 271.1 \text{ Hz}$$

In Fig. 7, the measured acceleration curve of the second mass,  $\ddot{x}_{2\max}$ , is shown as a function of frequency for a testing condition for which the control system was preset to provide an input acceleration to the first mass, ( $m_1$ ) of 0.5 g. The corresponding measured input signal



Notes:  $m_1$  - 12.42 Slugs, Manufacturer Listed Value

$m_2$  - 35.65 Slugs, Measured

$k_1$  -  $1.44 \times 10^5$  lb/ft, Manufacturer Listed Value

$k_2$  -  $2.62 \times 10^7$  lb/ft, Calculated Using a Modulus of Elasticity of  $3.0 \times 10^7$  for Steel Along with the Minimum Diameter and Nominal Length of the Threaded Studs

$F = F_0 \sin \omega_f t$  (Forcing Function)

Figure 5. Two-degree-of-freedom vibration system experiment.

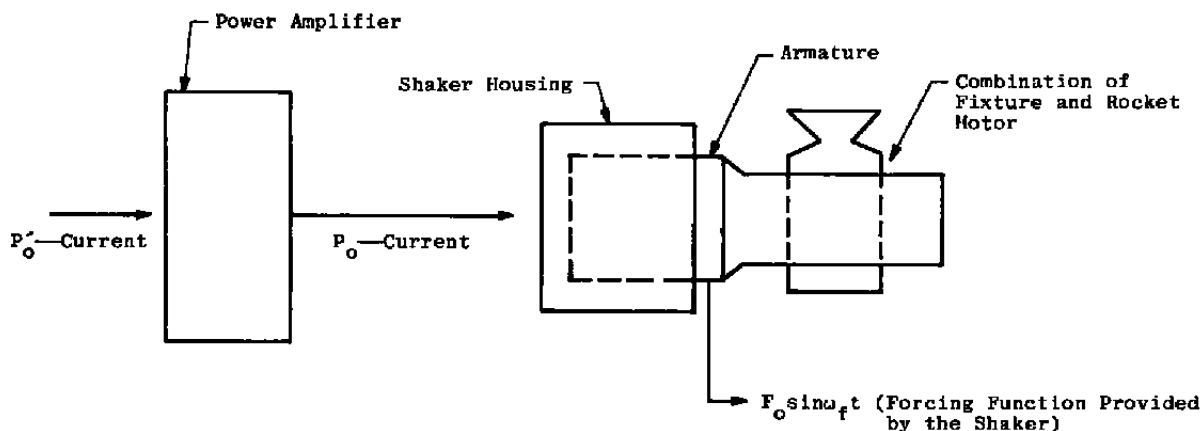
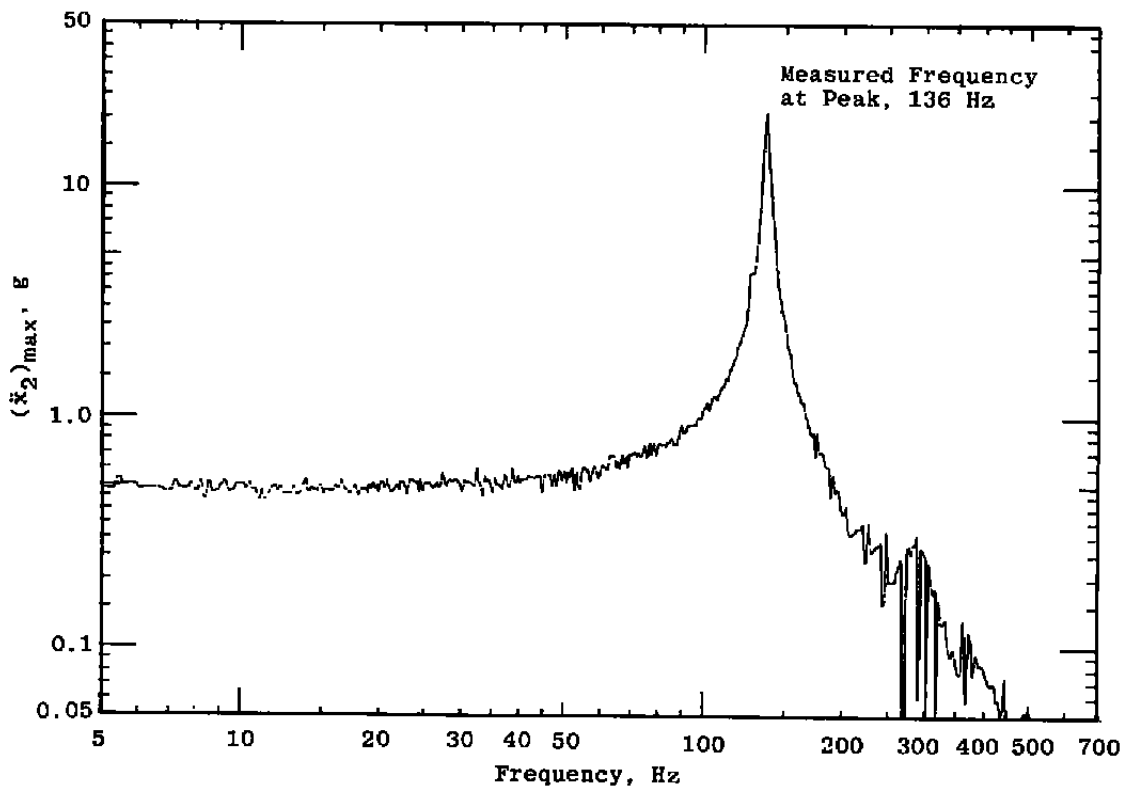


Figure 6.  $P_0$  and  $P'_0$  measurements.



**Figure 7. Acceleration response curve for the second mass of the two-degree-of-freedom system experiment.**

to the shaker,  $P_o$ , which is directly proportional to  $F_o$  (local maximum force applied by the shaker to the first mass) is shown in Fig. 8a. It is apparent that a very significant peak in  $(\ddot{x}_2)_{\max}$  occurs at the measured frequency of 136 Hz, and the corresponding shaker input signal,  $P_o$ , also has a well-defined peak at the measured frequency of 136 Hz. Note that the two measured frequencies corresponding to the peaks in the  $(\ddot{x}_2)_{\max}$  and  $P_o$  curves agree with the calculated frequency for  $\omega_2$  of the experimental system as would be expected. It follows that the measurements for the near-ideal two-DOF experimental system are consistent with the corresponding motion equations, and that at  $\omega_f = \omega_2$  (the natural frequency of the decoupled second mass), the natural behavior of such a system is to cause a significant increase in  $(\ddot{x}_2)_{\max}$  and a corresponding significant increase in  $P_o$ . Further, observe in Fig. 8a that the  $P_o$  curve has two well-defined local minimums at  $\omega_f$  values of 8.5 and 266 Hz. As expected, these frequencies agree with the calculated natural frequencies of the two-DOF system,  $\omega'_1$  and  $\omega'_2$ , listed above. It should be noted that the capability in computing  $\omega_2$ ,  $\omega'_1$ , and  $\omega'_2$ , for the experimental system is restricted by the limitation in defining the value of  $k_2$  for the threaded studs used.

The corresponding  $P'_o$  parameter is shown in Fig. 8b and is related to the input signal to the shaker,  $P_o$ ; again  $P_o$  is the measured input signal to the shaker, whereas  $P'_o$  is the measured input signal to the corresponding shaker power amplifier. From a comparison of

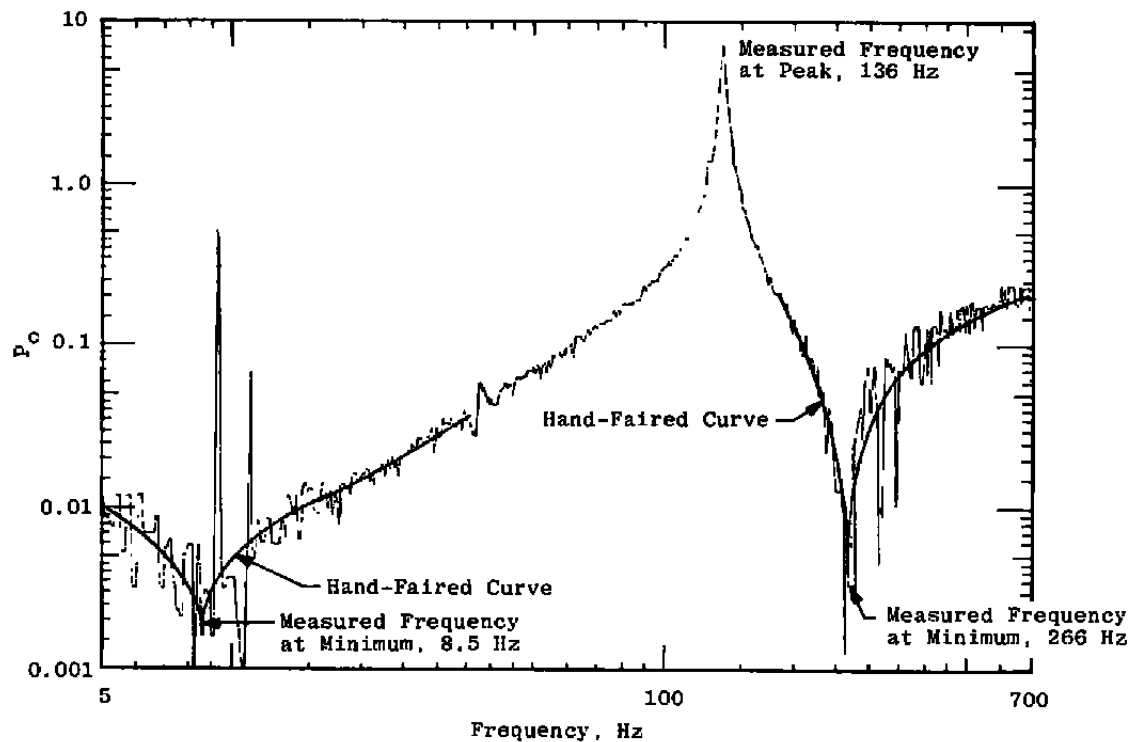
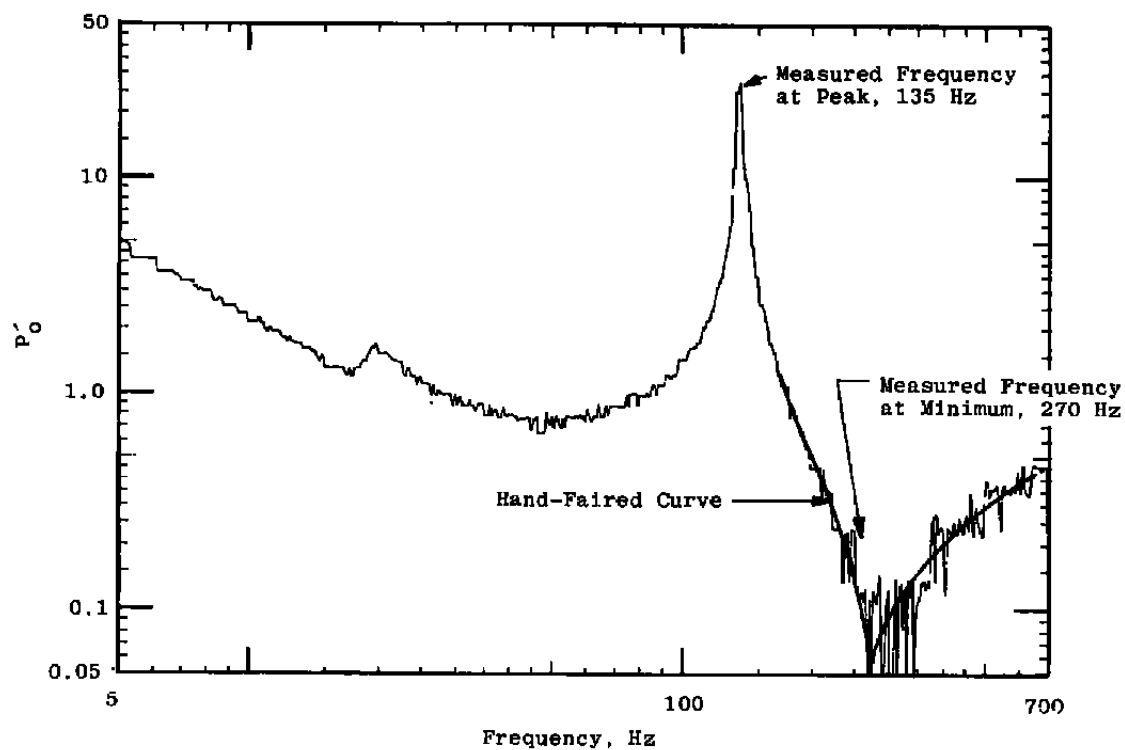
a. Shaker input signal,  $P_o$ b. Power amplifier input signal,  $P'_o$ 

Figure 8. Shaker inputs for the two-degree-of-freedom system experiment.



figures, 8a and 8b, it is apparent that the  $P'_0$  curve is quite adequate at frequencies above about 100 Hz for defining the basic trends associated with the  $P_0$  curve, whereas at the lower frequencies the  $P'_0$  measurements are invalid as they include significant power amplifier effects. The consistencies in the  $P'_0$  and  $P_0$  variations above 100 Hz are noted because in previous test programs, only  $P'_0$  measurements were available; in more recent tests,  $P_0$  measurements have been obtained as a standard procedure.

## 2.5 ROCKET MOTOR MEASUREMENTS

It has been well recognized that rocket motor test fixtures can behave adversely above some frequency level because of resonant characteristics inherent in such structures. Hence, it is of much importance to be able to define the frequency level above which such effects are nontrivial. In Fig. 9, the input acceleration  $(\ddot{x})_{\max}$  for a typical fixture-armature combination, is shown as a function of frequency. This curve indicates for frequencies up to about 500 Hz, except for a couple of negative spikes at frequencies of about 280 to 350 Hz, that the test fixture behaves quite well. However, in Fig. 10a the corresponding  $P_0$  curve is shown which indicates that the control system required appreciable excursions in the shaker input signal, starting at about 250 Hz. Such required excursions in  $P_0$ , though  $(\ddot{x})_{\max}$  was controlled reasonably well, indicate a tendency of the fixture for adverse behavior, and hence, the use

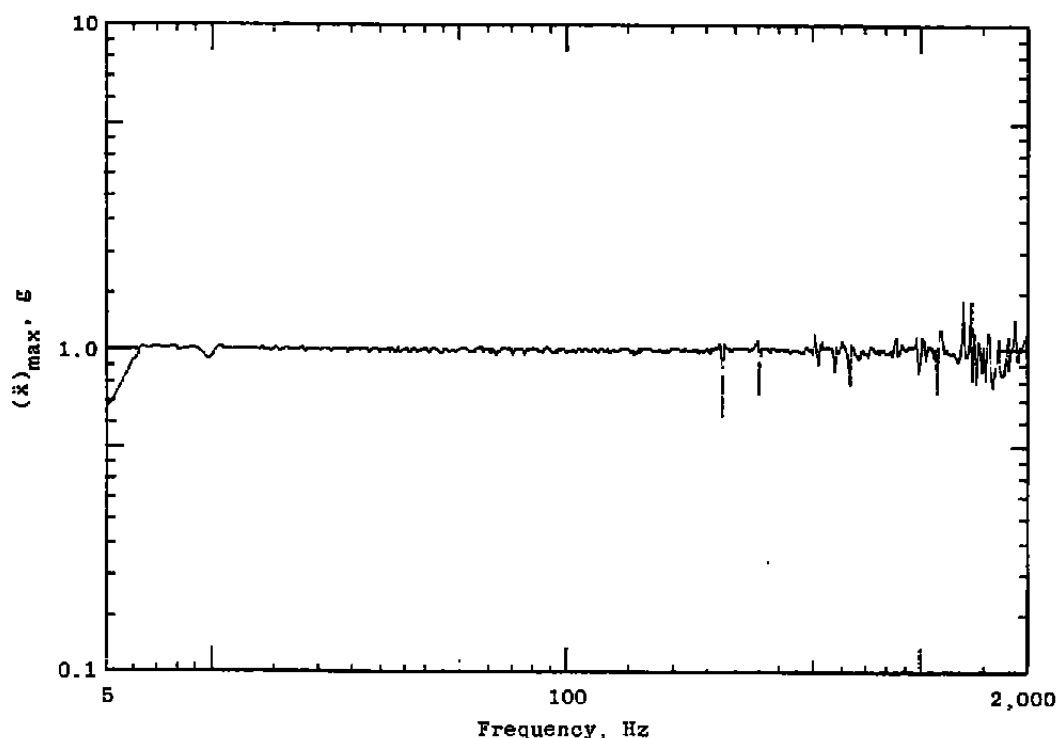
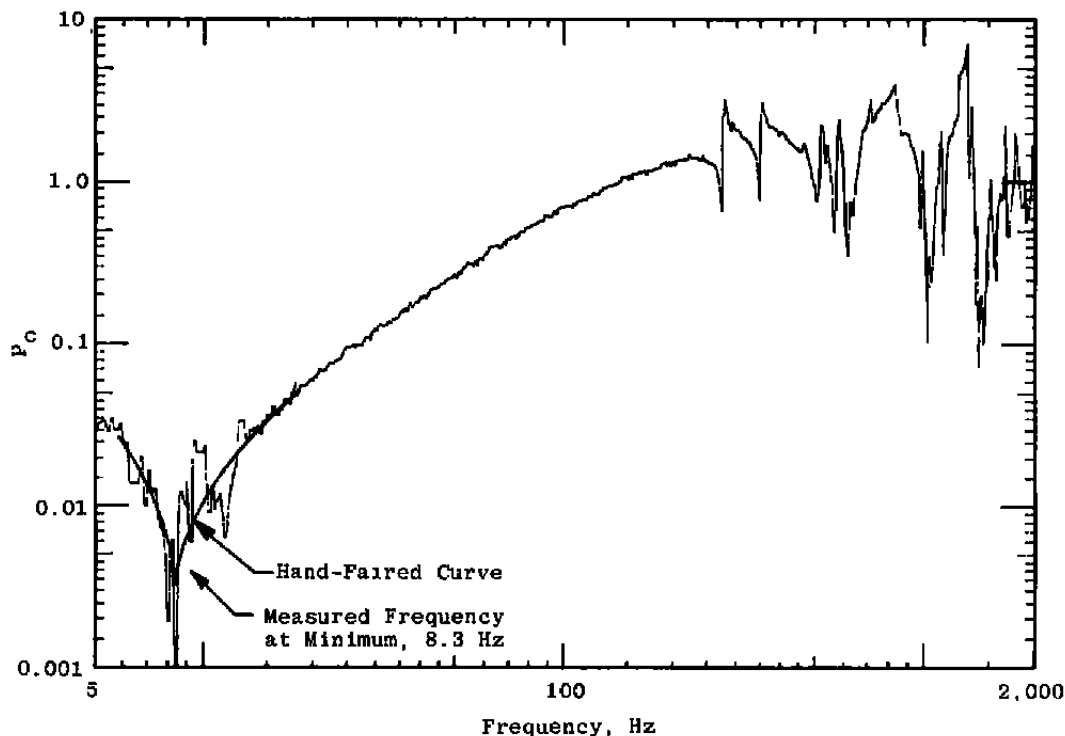


Figure 9. Acceleration curve for a typical armature-fixture combination.

of the test fixture above 250 Hz could be questionable. It follows that  $P_o$  is a sensitive parameter to test fixture behavior and can be expected to provide a reasonably good means for detection of the initiation of any adverse test fixture behavior. The corresponding  $P_o'$  curve is shown in Fig. 10b to provide further evidence of its agreement with  $P_o$  measurements at the higher frequencies. The  $P_o'$  curve for the armature-fixture combination that was used in the A-test program (see Fig. 2) is presented in Fig. 11a. This curve indicates that the test fixture used in that test program had no significant adverse motion below a frequency of about 200 Hz.

The measured  $P_o'$  curve corresponding to the rocket motor  $(\ddot{x}_2)_{\max}$  curve of Fig. 2 is shown in Fig. 11b;  $P_o$  measurements were not available in the A-test program. Note that the peak in the  $P_o'$  curve measured at a frequency of 141 Hz is consistent with the peak in the  $(\ddot{x}_2)_{\max}$  curve at 141 Hz in Fig. 2. These peaks observed in the  $(\ddot{x}_2)_{\max}$  and  $P_o'$  curves of Figs. 2 and 11b, in consideration of the discussion of the motion equations of Section 2.3 and the experimental data of Section 2.4, indicate that the rocket motor vibration system (Fig. 2) behaved predominantly as a two-DOF system at least up through a forcing frequency corresponding to the peaks in the  $(\ddot{x}_2)_{\max}$  and  $P_o'$  curves. This is a very important observation as the forcing frequency corresponding to the peaks in the  $(\ddot{x}_2)_{\max}$  and  $P_o'$  curves is  $\omega_2$  which,



a.  $P_o$

Figure 10. Shaker inputs for armature-fixture test of Fig. 9.

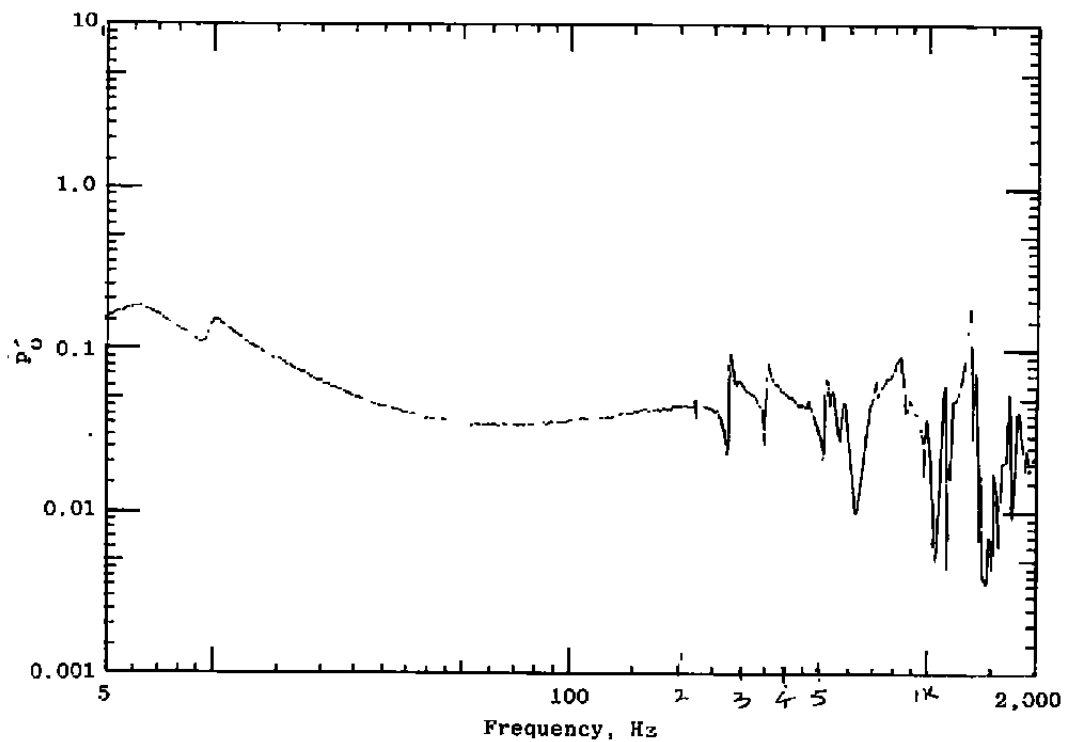
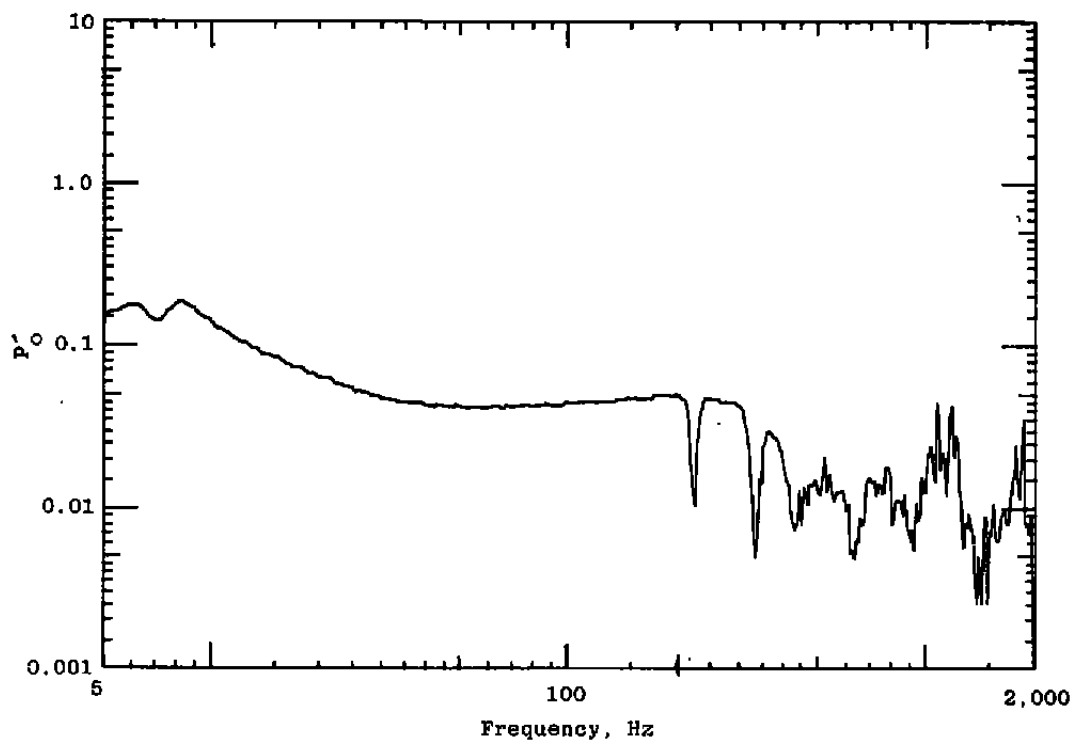
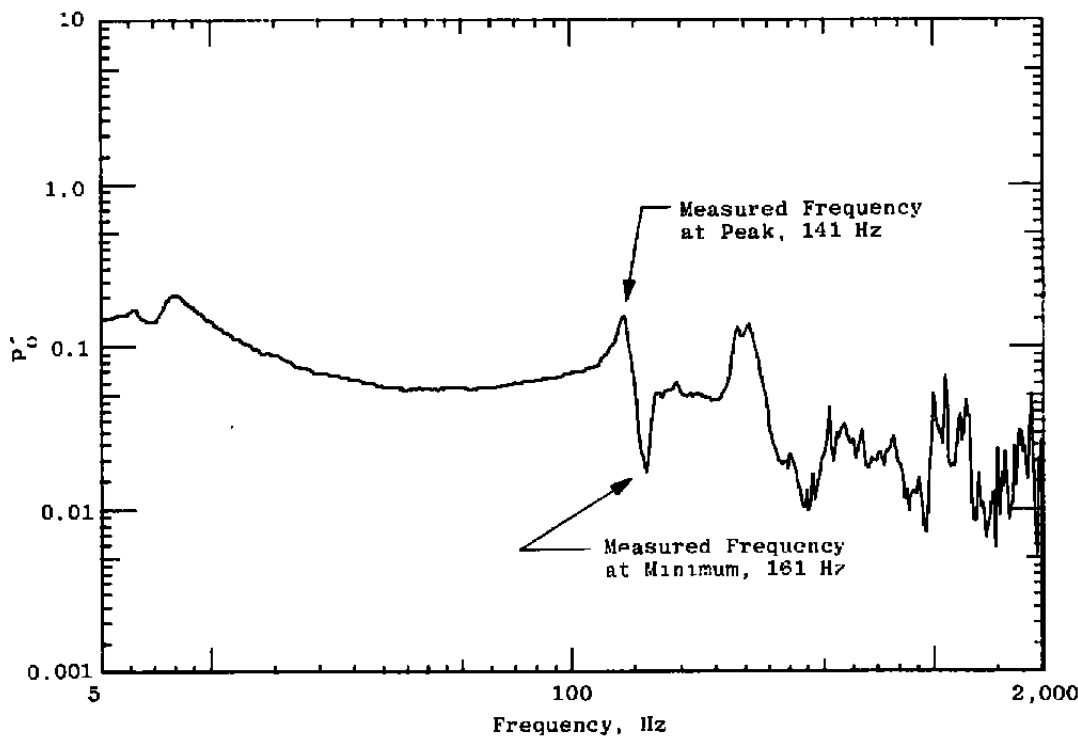
b.  $P'_0$ 

Figure 10. Concluded.



a. Armature-fixture combination

Figure 11. Inputs corresponding to the acceleration response curve of Fig. 2.



**b. Complete rocket motor system**

**Figure 11. Concluded.**

again, is the natural frequency of the uncoupled second mass (rocket motor). As discussed previously, an increase in  $(\dot{x}_2)_{\max}$  at  $\omega_f = \omega_2$  is a basic characteristic of a forced two-DOF system.

Considering the current analysis, the increase in  $(\dot{x}_2)_{\max}$  observed for the rocket motor in Fig. 2 (A-test program) would be expected. It follows then that the large increase in the acceleration response of the rocket motor at  $\omega_f \approx 141$  Hz (Fig. 2) should not be attributed to adverse test fixture effects.

Although there is a well-defined local minimum in the  $P'_0$  curve of Fig. 11b, consistent with the motion equations of Section 2.3, at a frequency that agrees well with a calculated  $\omega'_2$  (using an  $\omega_2$  value defined by peaks in the  $(\dot{x}_2)_{\max}$  and  $P'_0$  curves), this will not be the case, in general, in rocket motor vibration tests. Usually,  $\omega'_2$  will be from 10 to 50 percent larger than  $\omega_2$  and, hence, nearer to a frequency level involving nontrivial fixture effects. A testing condition having possible small fixture effects combined with the control system operating at a low-level force requirement (resonant condition) can produce a meaningless variation in the shaker input in the  $\omega'_2$  frequency range. It should also be noted that the apparent large noise level in some of the shaker input measurements is related to the low signal levels involved. Further, damping effects ignored in the present analysis would not be expected to have any significant effects on the results of the analysis.

### 3.0 CONCLUDING REMARKS

Results of an analysis of rocket motor acceleration response measurements from vibration tests indicate that large, undesirable motor accelerations may be measured and invalidly attributed to the test fixture used. Further, the use of the measured input signal to the shaker in forced vibration tests of rocket motors (not normally measured in the past) can be very useful in identifying the forcing frequency level above which significant adverse effects of a given test fixture can be expected.

### NOMENCLATURE

$F$	Forcing function, $F_0 \sin \omega_f t$
$F_0$	$F_{\max}$
$k_1$	Stiffness parameter of the $m_1$ mass
$k_2$	Stiffness parameter of the $m_2$ mass
$m_1$	First mass in a two-degree-of-freedom system
$m_2$	Second mass in a two-degree-of-freedom system
$P_0$	Shaker input current
$P'_0$	Power amplifier input current
$X_0$	Static displacement of the $m_1$ mass, $(F_0/k_1)$
$X_1$	$(x_1)_{\max}$
$X_2$	$(x_2)_{\max}$
$x_1$	Displacement of the $m_1$ mass
$x_2$	Displacement of the $m_2$ mass
$\ddot{x}_1, \ddot{x}_2$	Accelerations in the $x_1$ and $x_2$ directions, respectively
$\omega_f$	Forcing frequency

$\omega_1$	Natural frequency of the decoupled $m_1$ mass
$\omega_2$	Natural frequency of the decoupled $m_2$ mass
$\omega_1'$	First natural frequency of a two-degree-of-freedom system
$\omega_2'$	Second natural frequency of a two-degree-of-freedom system